IFT-P.022/96

Manifest Electromagnetic Duality in Closed Superstring Field Theory

Nathan Berkovits

Instituto de Física Teórica, Univ. Estadual Paulista
Rua Pamplona 145, São Paulo, SP 01405-900, BRASIL

e-mail: nberkovi@snfma2.if.usp.br

The free action for massless Ramond-Ramond fields is derived from closed superstring field theory using the techniques of Siegel and Zwiebach. For the uncompactified Type IIB superstring, this gives a manifestly Lorentz-covariant action for a self-dual five-form field strength. Upon compactification to four dimensions, the action depends on a U(1) field strength from 4D N=2 supergravity. However, unlike the standard Maxwell action, this action is manifestly invariant under the electromagnetic duality transformation which rotates F_{mn} into $\epsilon_{mnpq}F^{pq}$.

July 1996

1. Introduction

Due to recent conjectures regarding non-perturbative duality symmetries [1][2], there has been an increase of interest in the Ramond-Ramond sector of the superstring. However, problems caused by picture-changing operators have until now prevented a string field theory analysis of the Ramond-Ramond contribution to the action, even for the quadratic term.[3]

This is especially obvious for the Type IIB superstring, whose massless spectrum contains a self-dual five-form field strength for which it is difficult to construct a manifestly covariant action[4].[1] Also, there are R-R fields which allow more than one action (e.g., using a gauge field with P indices or with $D-P-2$ indices), and it is impossible to choose the correct action without an off-shell string field theory description.

In this paper, I begin by defining the R-R contribution to the free action using a version of superstring field theory which does not require picture-changing operators in either the NS or R sectors. This version is based on the open superstring field theory developed in reference [9], where it was shown that by adding a non-minimal set of variables to the usual RNS variables, both the NS and R contributions to the free open superstring action could be written as $\langle\Phi|Q|\Phi\rangle$. Generalization to closed superstring field theory is completely straightforward.

I then analyze the massless R-R sector using the techniques of reference [10] by Siegel and Zwiebach. Unlike the massless NS-NS sector, one has an infinite number of R-R fields due to the presence of bosonic ghost zero modes. However, the free action for these infinite fields can easily be written in closed form. For the Type IIB superstring, this gives a manifestly Lorentz-covariant action for a self-dual five-form field strength.[2]

After compactifying to four dimensions, one of the massless R-R fields is the U(1) field strength F^{mn} of 4D N=2 supergravity. Although one might expect an action of the form $\int d^4x F_{mn}F^{mn}$, compactification of the ten-dimensional action gives a completely different

[1] After breaking manifest Lorentz covariance, an action can be constructed for a self-dual five-form field strength.[5][6] Using the methods of reference [7], it is possible to "covariantize" this action by adding harmonic-like fields.[8]

[2] In Hamiltonian language, the self-duality condition involves second-class constraints. By introducing an infinite set of fields, these second-class constraints can be replaced with first-class constraints, allowing a covariant expression for the path integral.[11] It would be interesting to compare this Hamiltonian approach with the Lagrangian approach of this paper.

action which involves infinite fields and is manifestly invariant under the electromagnetic duality transformation which rotates F_{mn} into $\epsilon_{mnpq}F^{pq}$.

In the conclusion of this paper, I discuss possible generalizations of this work.

2. Ramond-Ramond contribution to superstring field theory

In standard open superstring field theory, the inner product of two string fields vanishes unless the sum of the pictures of the string fields is -2. This comes from the background charge on a sphere which implies that $\langle c\partial c\partial^2 c\ e^{-2\phi}\rangle$ is non-zero. (Note that the picture P of a string field Φ is defined by $\oint dz(i\xi\eta+\partial\phi)|\Phi\rangle=P|\Phi\rangle$ where $P=\oint dz(i\xi\eta+\partial\phi)$ and the bosonic ghosts have been fermionized as $\gamma=\eta e^{\phi}$ and $\beta=\partial\xi e^{-\phi}$.[12])

For Neveu-Schwarz string fields, this is not a problem since the free action $\langle\Phi|Q|\Phi\rangle$ carries -2 picture if Φ is chosen to carry -1 picture. However for Ramond string fields (which must carry half-integer picture), it is clear that $\langle\Phi|Q|\Phi\rangle$ cannot carry -2 picture. For open superstring field theory, a proposed solution was to modify the Ramond kinetic operator to $Q\ Y$ where $Y=c\partial\xi e^{-2\phi}$ is the inverse picture-changing operator.[13] Since Y carries -1 picture, the action $\langle\Phi|QY|\Phi\rangle$ carries -2 picture if the Ramond string field is chosen to carry $-\frac{1}{2}$ picture.

Although there are probably gauge-fixing problems at massive levels with the $\langle\Phi|QY|\Phi\rangle$ action [14], it at least gives the correct kinetic term for the massless Majorana-Weyl spinor,

$$\mathcal{S}=\int d^{10}x(is^{\alpha}\partial_{\alpha\beta}s^{\beta}) \tag{2.1}$$

where $\partial_{\alpha\beta}=\Gamma^{\mu}_{\alpha\beta}\partial_{\mu}$ and $\Gamma^{\mu}_{\alpha\beta}$ is the symmetric ten-dimensional σ-matrix. However, there have been no succesful generalizations of this action to the Ramond-Ramond sector of the closed superstring. The obvious guess,

$$\langle\Phi|(Q_L+Q_R)Y_LY_R(c_{0L}-c_{0R})|\Phi\rangle, \tag{2.2}$$

has the problem that Y_LY_R does not commute with $b_{0L}-b_{0R}$. (Throughout this paper, L and R subscripts refer to left and right-moving variables). This means that the action of (2.2)is not invariant under the gauge transformation $\delta|\Phi\rangle=(b_{0L}-b_{0R})(Q_L+Q_R)|\Lambda\rangle$, even when Φ satisfies the standard closed-string restrictions $(b_{0L}-b_{0R})|\Phi\rangle=(L_{0L}-L_{0R})|\Phi\rangle=0$.[3]

In reference [9], it was shown that by adding a new non-minimal set of variables to the usual RNS variables, both the NS and R contributions to the free open superstring field theory action could be written as $\langle\Phi|Q|\Phi\rangle$. These new non-minimal variables consist of a pair of conjugate bosons $(\tilde\gamma,\tilde\beta)$ of weight $(-\frac{1}{2},\frac{3}{2})$, and a pair of conjugate fermions (χ,u) of weight $(-\frac{1}{2},\frac{3}{2})$. The BRST operator is modified to $Q_{new}=Q_{RNS}+\int d\sigma\, u\tilde\gamma$, so using the standard "quartet" argument, the new non-minimal fields do not affect the physical cohomology. Like the ψ^μ matter fields and (γ,β) ghost fields, $(\tilde\gamma,\tilde\beta)$ and (χ,u) are defined to be odd under G-parity.

It will be convenient to fermionize (γ,β) and $(\tilde\gamma,\tilde\beta)$ in the following non-standard way:

$$t^+=\gamma+i\tilde\gamma=\eta e^{\phi},\quad \bar t^+=\gamma-i\tilde\gamma=\bar\eta e^{\bar\phi}, \tag{2.3}$$

$$t^-=\frac{1}{2}(\beta+i\tilde\beta)=\partial\bar\xi e^{-\bar\phi},\quad \bar t^-=\frac{1}{2}(\beta-i\tilde\beta)=\partial\xi e^{-\phi},$$

where

$$t^j(y)\bar t^k(z)\to\epsilon^{jk}(y-z)^{-1},\qquad (\epsilon^{+-}=-\epsilon^{-+}=1)$$

$$\phi(y)\phi(z)\to-\log(y-z),\quad \bar\phi(y)\bar\phi(z)\to-\log(y-z),$$

$$\eta(y)\xi(z)\to i(y-z)^{-1},\quad \bar\eta(y)\bar\xi(z)\to i(y-z)^{-1}$$

are the singular OPE's as $y\to z$.

There are now two picture operators,

$$P=\oint dz(i\xi\eta+\partial\phi),\quad \bar P=\oint dz(i\bar\xi\bar\eta+\partial\bar\phi), \tag{2.4}$$

which are complex conjugates of each other. One can therefore choose the "in" Ramond string field, $|\Phi\rangle$, to carry picture $P=-\frac{1}{2}$ and $\bar P=-\frac{3}{2}$, which implies that the "out" Ramond string field, $\langle\Phi|$, carries $\bar P=-\frac{1}{2}$ and $P=-\frac{3}{2}$. (For NS string fields, both the "in" and "out" fields carry picture $P=\bar P=-1$.) Since the total picture for both P and $\bar P$ is -2, the kinetic operator no longer needs to carry picture and the action is simply $\langle\Phi|Q|\Phi\rangle$. Note that the background charge now implies that $\langle c\partial c\partial^2 c\ u\partial u\ e^{-2(\phi+\bar\phi)}\rangle$ is non-zero.

As explained in reference [9], the reality conditions on Φ are slightly unusual. Normally, one requires that hermitian conjugation is equivalent (up to a sign) with BPZ conjugation.[15] In this case, one instead requires that hermitian conjugation is equivalent

(up to a sign) with BPZ conjugation times "charge conjugation", where "charge conjugation" simply flips the sign of all non-minimal fields (i.e. $[\tilde{\gamma}, \tilde{\beta}, u, \chi] \to [-\tilde{\gamma}, -\tilde{\beta}, -u, -\chi]$). Note that charge conjugation commutes with the BRST operator.

To generalize to the closed superstring, one simply introduces both left-moving and right-moving non-minimal fields: $(\tilde{\gamma}_L, \tilde{\beta}_L,\ u_L, \chi_L)$ and $(\tilde{\gamma}_R, \tilde{\beta}_R,\ u_R, \chi_R)$. One then defines the Ramond-Ramond "in" string field, $|\Phi\rangle$, to have picture $[P_L, \bar{P}_L, P_R, \bar{P}_R]$ $=[-\frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}, -\frac{3}{2}]$, and the Ramond-Ramond "out" string field, $\langle\Phi|$, to have picture $[P_L, \bar{P}_L, P_R, \bar{P}_R]$ $=[-\frac{3}{2}, -\frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}]$. As usual for closed string fields, Φ must also satisfy $(b_{0L} - b_{0R})|\Phi\rangle= (L_{0L} - L_{0R})|\Phi\rangle = 0$.[15]After imposing these restrictions, the action is simply

$$\mathcal{S} = \langle\Phi|(Q_L + Q_R)(c_{0L} - c_{0R})|\Phi\rangle \tag{2.5}$$

where $Q_L + Q_R$ includes the non-minimal term $\int d\sigma(\tilde{\gamma}_L u_L + \tilde{\gamma}_R u_R)$.

3. Massless Ramond-Ramond contribution to the free action

To analyze the component form of (2.5), it is convenient to use the SU(1,1) formalism developed by Siegel and Zwiebach in reference [10]. Although this SU(1,1) method has not been generalized to include interactions, if one only wants the quadratic term, it is much simpler than directly performing a component expansion of (2.5). Before using this method to derive the massless Ramond-Ramond contribution to the free closed superstring action, I will first review the massless Ramond contribution to the free open superstring action.[9]

3.1. The open superstring

For an open string action of the form $\langle\Phi|Q|\Phi\rangle$, the first step in the SU(1,1) method is to define T^{++} and Q^{+} by

$$Q = c_0 L_0 + b_0 T^{++} + Q^{+} \tag{3.1}$$

where Q^{+} includes all terms in Q which are independent of the fermionic ghost zero modes, b_0 and c_0. The next step is to define T^{--} and T^{+-} in such a way that $[T^{++}, T^{+-}, T^{--}]$ generate an SU(1,1) algebra. For example, for the bosonic open string,

$$T^{++} = \sum_{n>0} 2n c_n c_{-n}, \quad T^{+-} = \frac{1}{2}\sum_{n>0} i(c_n b_{-n} - b_n c_{-n}), \quad T^{--} = \sum_{n>0} \frac{1}{2n} b_n b_{-n}. \tag{3.2}$$

(This algebra is SU(1,1) rather than SU(2) since $(T^{++})^\dagger = T^{++}$ rather than $(T^{++})^\dagger = T^{--}$.)

Note that $2T^{+-} + ic_0 b_0$ is the ghost-number operator and nilpotence of Q implies that $[Q^+, T^{++}] = 0$ and $(Q^+)^2 = -T^{++} L_0$. Therefore, Q^+ transforms under SU(1,1) as the top component of a two-component spinor Q^j ($j = \pm$) which satisfies

$$\{Q^j, Q^k\} = -2T^{jk} L_0. \tag{3.3}$$

In reference [10], Siegel and Zwiebach show that any $\langle\Phi|Q|\Phi\rangle$ action can equivalently be written as

$$\langle\varphi|c_0(2L_0 + iQ^j Q_j)|\varphi\rangle \tag{3.4}$$

where $Q_j = \epsilon_{jk} Q^k$ and φ is an SU(1,1) singlet which is independent of c_0 (i.e. $T^{jk}|\varphi\rangle = b_0|\varphi\rangle = 0$). This action has the gauge invariance $\delta|\varphi\rangle = Q^j|\Lambda_j\rangle$ where Λ_j is an SU(1,1) doublet satisfying $b_0|\Lambda_j\rangle = 0$. Because one only needs to consider string fields which are SU(1,1) singlets, it is much easier to perform a component analysis of (3.4)than of $\langle\Phi|Q|\Phi\rangle$.

For the massless Ramond contribution to the free open superstring action, only the zero modes in Q are relevant (the positive modes all annihilate the massless states):

$$Q = c_0 p^2 - b_0 \gamma_0^2 + \gamma_0 \psi_0^\mu p_\mu + \tilde{\gamma}_0 u_0 \tag{3.5}$$

where $p_\mu = i\partial_\mu$ is the momentum and $\{\psi_0^\mu, \psi_0^\nu\} = 2\eta^{\mu\nu}$. Before applying the SU(1,1) method, it is convenient to first perform the unitary transformation

$$Q \to e^{\chi_0 b_0 \tilde{\gamma}_0} Q e^{-\chi_0 b_0 \tilde{\gamma}_0}, \tag{3.6}$$

which transforms Q into

$$\begin{aligned} Q &= (c_0 + \chi_0\tilde{\gamma}_0)p^2 - b_0(\gamma_0^2 + \tilde{\gamma}_0^2) + \gamma_0\psi_0^\mu p_\mu + \tilde{\gamma}_0 u_0 \\ &= c_0 p^2 - b_0 t_0^+ \bar{t}_0^+ + \frac{1}{2}(t_0^+ + \bar{t}_0^+)\psi_0^\mu p_\mu - \frac{i}{2}(t_0^+ - \bar{t}_0^+)(u_0 + \chi_0 p^2) \end{aligned} \tag{3.7}$$

where t^j is defined in (2.3). For notational convenience, I will supress the 0 subscript when it is obvious.

So using the definitions of (3.1),

$$L = p^2, \quad T^{jk} = -\frac{1}{2}(t^j \bar{t}^k + t^k \bar{t}^j), \tag{3.8}$$

$$Q^j = \frac{1}{2}(t^j + \bar{t}^j)\psi^\mu p_\mu - \frac{i}{2}(t^j - \bar{t}^j)(u + \chi p^2).$$

Note that $[t^j, \bar{t}^k] = i\epsilon^{jk}$.

For the "in" string field $|\varphi\rangle$ which has picture $[P, \bar{P}] = [-\frac{1}{2}, -\frac{3}{2}]$, the massless states which are independent of c_0 can be written as

$$|\varphi\rangle = f^\alpha(x^\mu, \psi^\mu, iu, t^+, t^-)|0_\alpha\rangle$$

where $|0_\alpha\rangle$ is an SO(9,1) Majorana-Weyl spinor whose vertex operator in terms of the super-SL_2 invariant vacuum is

$$V = ce^{-\frac{1}{2}(\phi + 3\bar{\phi} + i\sigma + \frac{i\pi}{2})}S_\alpha. \tag{3.9}$$

($u = e^{-i\sigma}$, $\chi = e^{i\sigma}$, and S_α is the usual spin field of weight $\frac{5}{8}$.) Note that $\bar{t}_0^j|0_\alpha\rangle = \chi_0|0_\alpha\rangle = 0$.

The reality condition discussed earlier implies that f^α is a real function (which is the reason for the $e^{-\frac{i\pi}{4}}$ factor in (3.9)). Furthermore, the GSO projection restricts f^α to have even G-parity, where ψ^μ, u, and t^j all carry odd G-parity. Finally, the SU(1,1) singlet condition, $T^{jk}|\varphi\rangle = 0$, implies that f^α is independent of t^j, and therefore,

$$|\varphi\rangle = s^\alpha(x)|0_\alpha\rangle + u\kappa_\beta(x)|0^\beta\rangle \tag{3.10}$$

where $\psi^\mu|0_\alpha\rangle = \Gamma^\mu_{\alpha\beta}|0^\beta\rangle$. (There is no factor of i in (3.10)since $|0^\beta\rangle$ is fermionic, and therefore $(u\kappa_\beta|0^\beta\rangle)^\dagger = -u\kappa_\beta\langle 0^\beta|$, which goes to $u\kappa_\beta|0^\beta\rangle$ under charge conjugation.)

Plugging into (3.4), one finds

$$\mathcal{S} = \langle\varphi|c_0(2L_0 + iQ^jQ_j)|\varphi\rangle \tag{3.11}$$

$$= \langle\varphi|c(p^2 + \frac{1}{2}(t^j + \bar{t}^j)(t_j - \bar{t}_j)p_\mu\psi^\mu(u + \chi p^2))|\varphi\rangle,$$

where $\langle\varphi| = \langle 0_\alpha|s^\alpha(x) + \langle 0^\beta|\kappa_\beta(x)u$ and the vertex operator for $\langle 0_\alpha|$ is

$$V = ce^{-\frac{1}{2}(\bar{\phi} + 3\phi + i\sigma - \frac{i\pi}{2})}S_\alpha. \tag{3.12}$$

Using the normalization condition that $\langle 0_\alpha|c_0u_0\ h(x)|0^\beta\rangle = i\delta_\alpha^\beta \int d^{10}xh(x)$, it is easy to compute that

$$\mathcal{S} = \int d^{10}x(-2is^\alpha\partial_\mu\partial^\mu\kappa_\alpha + is^\alpha\partial_{\alpha\beta}s^\beta - i\kappa_\alpha\partial^{\alpha\beta}\partial_\mu\partial^\mu\kappa_\beta) \tag{3.13}$$

$$= \int d^{10}x(i\hat{s}^\alpha\partial_{\alpha\beta}\hat{s}^\beta)$$

where $\hat{s}^\alpha = s^\alpha - \partial^{\alpha\beta}\kappa_\beta$. Note that (3.13)is invariant under the gauge transformation:

$$\delta|\varphi\rangle = Q^j(-t_j\Lambda_\beta)|0^\beta\rangle = u\Lambda_\beta|0^\beta\rangle + \partial^{\alpha\beta}\Lambda_\beta|0_\alpha\rangle, \tag{3.14}$$

i.e. $\delta\kappa_\beta = \Lambda_\beta$, $\delta s^\alpha = \partial^{\alpha\beta}\Lambda_\beta$.

3.2. The closed superstring

For the closed superstring, the SU(1,1) method is generalized in the obvious way. One uses Q_L and Q_R to define T_L^{jk}, Q_L^j, T_R^{jk}, and Q_R^j, and the action is

$$\langle\varphi|c_{0L}c_{0R}(2L_{0L}+i(Q_L^j+Q_R^j)(Q_{jL}+Q_{jR}))|\varphi\rangle \tag{3.15}$$

where $|\varphi\rangle$ satisfies

$$b_{0L}|\varphi\rangle = b_{0R}|\varphi\rangle = (L_{0L}-L_{0R})|\varphi\rangle = (T_L^{jk}+T_R^{jk})|\varphi\rangle = 0. \tag{3.16}$$

This action has the gauge invariance

$$\delta|\varphi\rangle = (Q_L^j+Q_R^j)|\Lambda_j\rangle \tag{3.17}$$

where $b_{0L}|\Lambda_j\rangle = b_{0R}|\Lambda_j\rangle = 0$ and Λ_j transforms as an SU(1,1) doublet under $T_L^{jk}+T_R^{jk}$.

For the "in" closed-string field with picture $[P_L,\bar P_L,\, P_R,\bar P_R]$= $[-\frac{1}{2},-\frac{3}{2},\,-\frac{1}{2},-\frac{3}{2}]$, the massless states of the Type IIB superstring can be written as

$$|\varphi\rangle = f^{\alpha\beta}(x^\mu,\psi_L^\mu,\psi_R^\mu,iu_L,iu_R,t_L^j,t_R^j)|L_\alpha\rangle|R_\beta\rangle \tag{3.18}$$

where $|L_\alpha\rangle$ and $|R_\beta\rangle$ are defined like $|0_\alpha\rangle$ in the open superstring ($|L_\alpha\rangle$ is left-moving and $|R_\beta\rangle$ is right-moving). $f^{\alpha\beta}$ is real and, for the Type IIB superstring, has even left and right-moving G-parity. (For the Type IIA superstring, $f^{\alpha\beta}$ has even left-moving and odd right-moving G-parity). Furthermore, the SU(1,1) singlet condition, $(T_L^{jk}+T_R^{jk})|\varphi\rangle=0$, implies that $f^{\alpha\beta}$ depends on t_L^j and t_R^j only in the combination $\epsilon_{jk}t_L^jt_R^k$. Therefore, for Type IIB,

$$\begin{aligned}|\varphi\rangle = \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}(it_L^jt_{jR})^{2n}(&F_{(2n)}^{\alpha\beta}|L_\alpha\rangle|R_\beta\rangle + u_LE^\beta_{(2n)\alpha}|L^\alpha\rangle|R_\beta\rangle\\ &+u_RD^\alpha_{(2n)\beta}|L_\alpha\rangle|R^\beta\rangle + u_Lu_RC_{(2n)\alpha\beta}|L^\alpha\rangle|R^\beta\rangle)\\ +\sum_{n=0}^{\infty}\frac{1}{(2n+2)!}(it_L^jt_{jR})^{2n+1}(&F_{(2n+1)\alpha\beta}|L^\alpha\rangle|R^\beta\rangle + u_LE^\alpha_{(2n+1)\beta}|L_\alpha\rangle|R^\beta\rangle\\ &+u_RD^\beta_{(2n+1)\alpha}|L^\alpha\rangle|R_\beta\rangle + u_Lu_RC^{\alpha\beta}_{(2n+1)}|L_\alpha\rangle|R_\beta\rangle).\end{aligned} \tag{3.19}$$

So at the massless level, an infinite number of fields are present in the Ramond-Ramond sector of the closed superstring. (For the Type IIA superstring, the only difference is in the position of the right-moving spinor index on F, E, D, and C.)

Plugging into (3.15)and keeping only the zero modes, one derives that the massless Ramond-Ramond contribution to the free action is:

$$\mathcal{S} = \langle\varphi|c_{0L}c_{0R}(2L_{0L} + i\epsilon_{jk}(Q^j_L + Q^j_R)(Q^k_L + Q^k_R))|\varphi\rangle \tag{3.20}$$

$$= \langle\varphi|c_L c_R((i - t^j_L\bar{t}_{jL})p_\mu\psi^\mu_L(u_L + \chi_L p^2) + (i - t^j_R\bar{t}_{jR})p_\mu\psi^\mu_R(u_R + \chi_R p^2)$$

$$+\frac{1}{2}((t^j_L + \bar{t}^j_L)(t_{jR} - \bar{t}_{jR})p_\mu\psi^\mu_L(u_R + \chi_R p^2) + (t^j_L - \bar{t}^j_L)(t_{jR} + \bar{t}_{jR})(u_L + \chi_L p^2)p_\mu\psi^\mu_R)$$

$$+\frac{i}{2}(t^j_L + \bar{t}^j_L)(t_{jR} + \bar{t}_{jR})p_\mu\psi^\mu_L p_\nu\psi^\nu_R$$

$$-\frac{i}{2}(t^j_L - \bar{t}^j_L)(t_{jR} - \bar{t}_{jR})(u_L + \chi_L p^2)(u_R + \chi_R p^2))|\varphi\rangle.$$

Finally, using the fact that

$$\langle R_\alpha|\langle L_\beta|(\bar{t}^j_L\bar{t}_{jR})^m(t^j_L t_{jR})^n\ c_{0L}c_{0R}\ u_{0L}u_{0R}\ h(x)|L^\gamma\rangle|R^\delta\rangle$$

$$= \delta_{n,m}\ n!(n+1)!\ (-1)^n\delta^\delta_\alpha\delta^\gamma_\beta\int d^{10}x h(x), \tag{3.21}$$

it is straightforward to compute that

$$\mathcal{S} = \sum_{n=0}^{\infty}[2C_{(n)\alpha\beta}\partial_\mu\partial^\mu(\partial^{\alpha\gamma}E^\beta_{(n)\gamma} + (-1)^n\partial^{\beta\gamma}D^\alpha_{(n)\gamma})$$

$$-2F^{\alpha\beta}_{(n)}((-1)^n\partial_{\beta\gamma}E^\gamma_{(n)\alpha} + \partial_{\alpha\gamma}D^\gamma_{(n)\beta})$$

$$-(F^{\alpha\beta}_{(n)} + \partial^{\alpha\gamma}E^\beta_{(n)\gamma} + (-1)^n\partial^{\beta\gamma}D^\alpha_{(n)\gamma} - (-1)^n\partial^{\alpha\gamma}\partial^{\beta\delta}C_{(n)\gamma\delta}) \tag{3.22}$$

$$(F_{(n+1)\alpha\beta} + (-1)^n\partial_{\beta\kappa}D^\kappa_{(n+1)\alpha} - \partial_{\alpha\kappa}E^\kappa_{(n+1)\beta} + (-1)^n\partial_{\alpha\kappa}\partial_{\beta\epsilon}C^{\kappa\epsilon}_{(n+1)})]$$

where it is understood that for odd n, the positions of all spinor indices are reversed (e.g. the first term for $n = 1$ is $2C^{\alpha\beta}_{(1)}\partial_\mu\partial^\mu\partial_{\alpha\gamma}E^\gamma_{(1)\beta}$).

Although (3.22)looks complicated, it is easy to analyze because of the gauge invariance:

$$\delta|\varphi\rangle = Q^j\sum_{n=0}^{\infty}\frac{1}{(n+2)!}(it^k_L t_{Rk})^n \tag{3.23}$$

$$(-2t_{jL}\Lambda^\beta_{(n)\alpha}|L^\alpha\rangle|R_\beta\rangle - 2t_{jR}\Omega^\alpha_{(n)\beta}|L_\alpha\rangle|R^\beta\rangle - 2t_{jL}u_R\Theta_{(n)\alpha\beta}|L^\alpha\rangle|R^\beta\rangle)$$

$$= \sum_{n=0}^{\infty}\frac{(it^j_L t_{jR})^n}{(n+1)!}(u_L u_R\Theta_{(n)\alpha\beta}|L^\alpha\rangle|R^\beta\rangle + u_R(\Omega^\alpha_{(n)\beta} - \partial^{\alpha\gamma}\Theta_{(n)\gamma\beta})|L_\alpha\rangle|R^\beta\rangle$$

$$+u_L\Lambda^{\beta}_{(n)\alpha}|L^{\alpha}\rangle|R_{\beta}\rangle+(\partial^{\alpha\gamma}\Lambda^{\beta}_{(n)\gamma}+(-1)^n\partial^{\beta\gamma}\Omega^{\alpha}_{(n)\gamma})|L_{\alpha}\rangle|R_{\beta}\rangle)$$

$$+\sum_{n=0}^{\infty}\frac{(it^j_L t_{jR})^{n+1}}{(n+2)!}(u_R(-\Lambda^{\beta}_{(n)\alpha}+(-1)^n\partial^{\beta\gamma}\Theta_{(n)\alpha\gamma})|L^{\alpha}\rangle|R_{\beta}\rangle+u_L\Omega^{\alpha}_{(n)\beta}|L_{\alpha}\rangle|R^{\beta}\rangle+$$

$$(-(-1)^n\partial_{\beta\gamma}\Lambda^{\gamma}_{(n)\alpha}-\partial_{\alpha\gamma}\Omega^{\gamma}_{(n)\beta}+\partial_{\mu}\partial^{\mu}\Theta_{(n)\alpha\beta})|L^{\alpha}\rangle|R^{\beta}\rangle)$$

where $\Lambda^{\beta}_{(n)\alpha}$, $\Omega^{\alpha}_{(n)\beta}$, and $\Theta_{(n)\alpha\beta}$ are independent fields (for odd n, the positions of their spinor indices are reversed).

Comparing with (3.19), this implies that (3.22)is gauge-invariant under

$$\delta C_{(n)\alpha\beta}=\Theta_{(n)\alpha\beta},\quad \delta D^{\alpha}_{(n)\beta}=\Omega^{\alpha}_{(n)\beta}-\partial^{\alpha\gamma}\Theta_{(n)\gamma\beta},\tag{3.24}$$

$$\delta E^{\beta}_{(n)\alpha}=\Lambda^{\beta}_{(n)\alpha},\quad \delta F^{\alpha\beta}_{(n)}=\partial^{\alpha\gamma}\Lambda^{\beta}_{(n)\gamma}+(-1)^n\partial^{\beta\gamma}\Omega^{\alpha}_{(n)\gamma},$$

$$\delta D^{\beta}_{(n+1)\alpha}=-\Lambda^{\beta}_{(n)\alpha}+(-1)^n\partial^{\beta\gamma}\Theta_{(n)\alpha\gamma},\quad \delta E^{\alpha}_{(n+1)\beta}=\Omega^{\alpha}_{(n)\beta},$$

$$\delta F_{(n+1)\alpha\beta}=-(-1)^n\partial_{\beta\gamma}\Lambda^{\gamma}_{(n)\alpha}-\partial_{\alpha\gamma}\Omega^{\gamma}_{(n)\beta}+\partial_{\mu}\partial^{\mu}\Theta_{(n)\alpha\beta}.$$

Therefore, $\Lambda^{\beta}_{(n)\alpha}$, $\Omega^{\alpha}_{(n)\beta}$, and $\Theta_{(n)\alpha\beta}$ can be used to gauge

$$E^{\beta}_{(n)\alpha}=D^{\alpha}_{(n)\beta}=C_{(n)\alpha\beta}=0\tag{3.25}$$

for all n. In this gauge,

$$|\varphi\rangle=\sum_{n=0}^{\infty}(\frac{(it^j_L t_{jR})^{2n}}{(2n+1)!}F^{\alpha\beta}_{(2n)}|L_{\alpha}\rangle|R_{\beta}\rangle+\frac{(it^j_L t_{jR})^{2n+1}}{(2n+2)!}F_{(2n+1)\alpha\beta}|L^{\alpha}\rangle|R^{\beta}\rangle),$$

and the equation of motion

$$(2L_{0L}+i(Q^j_L+Q^j_R)(Q_{jL}+Q_{jR}))|\varphi\rangle=0\tag{3.26}$$

implies that

$$0=F_{(1)\alpha\beta}=-F_{(3)\alpha\beta}=F_{(5)\alpha\beta}=-F_{(7)\alpha\beta}=...,\tag{3.27}$$

$$F^{\alpha\beta}_{(0)}=-F^{\alpha\beta}_{(2)}=F^{\alpha\beta}_{(4)}=-F^{\alpha\beta}_{(6)}=...,\tag{3.28}$$

$$\partial_{\alpha\gamma}F^{\alpha\beta}_{(0)}=\partial_{\beta\gamma}F^{\alpha\beta}_{(0)}=0.\tag{3.29}$$

Equation (3.27)comes from the $u_L u_R$ $(t^j_L t_{jR})^{2n}$ component of (3.26), equation (3.28)comes from the $u_L u_R$ $(t^j_L t_{jR})^{2n+1}$ component of (3.26), and equation (3.29)comes from the u_L and u_R components of (3.26).

So, on-shell, the only non-vanishing field in this gauge is $F^{\alpha\beta}_{(0)}$, which satisfies the equations of motion of (3.29). Expanding in vector notation,

$$F^{\alpha\beta}_{(0)} = \Gamma^{\alpha\beta}_{\mu} F^{\mu}_{(0)} + \Gamma^{\alpha\beta}_{\mu\nu\rho} F^{\mu\nu\rho}_{(0)} + \Gamma^{\alpha\beta}_{\mu\nu\rho\kappa\sigma} F^{\mu\nu\rho\kappa\sigma}_{(0)} \tag{3.30}$$

where $F^{\mu\nu\rho\kappa\sigma}_{(0)}$ is self-dual. Equation (3.29)implies that these one-form, three-form, and self-dual five-form field strengths obey

$$\partial^{[\nu} F^{\mu]}_{(0)} = \partial^{[\nu} F^{\mu\rho\kappa]}_{(0)} = 0 \tag{3.31}$$

$$\partial_{\mu} F^{\mu}_{(0)} = \partial_{\mu} F^{\mu\nu\rho}_{(0)} = \partial_{\mu} F^{\mu\nu\rho\kappa\sigma}_{(0)} = 0, \tag{3.32}$$

which are the usual Bianchi identities and equations of motion. (For the Type IIA superstring, one gets Bianchi identities and equations of motion for a two-form and a four-form field strength.)

It might seem puzzling that in the gauge of (3.25), the action of (3.22)becomes

$$\mathcal{S} = -\sum_{n=0}^{\infty} F^{\alpha\beta}_{(n)} F_{(n+1)\alpha\beta}, \tag{3.33}$$

which naively appears to have no propagating degrees of freedom. This paradox comes from the fact that there are an infinite number of fields and one needs to be careful when taking the limit $n \to \infty$.

The correct procedure is to first compute the equations of motion with a finite cutoff, so n ranges from 0 to N. If N is finite, one can gauge away $[E_{(n)}, D_{(n)}, C_{(n)}]$ for $n < N$, but cannot gauge away $[E_{(N)}, D_{(N)}, C_{(N)}]$ since the gauge transformations of (3.24)parameterized by $[\Lambda_{(N)}, \Omega_{(N)}, \Theta_{(N)}]$ also transform $[F_{(N+1)}, E_{(N+1)}, D_{(N+1)}, C_{(N+1)}]$. In this gauge, one gets the equations of motion of (3.27)(3.28)(3.29)for $n < N$, as well as non-trivial equations of motion for $[F_{(N)}, E_{(N)}, D_{(N)}, C_{(N)}]$. In the limit as $N \to \infty$, one can ignore $[F_{(N)}, E_{(N)}, D_{(N)}, C_{(N)}]$, so one is left with just the equations of motion of (3.27)(3.28)(3.29).

4. Manifest electromagnetic duality in four dimensions

After compactifying the Type II superstring on a generic six-dimensional Calabi-Yau manifold, the resulting superstring contains at least N=2 four-dimensional supersymmetry. For each massless ten-dimensional Ramond-Ramond field $F^{\alpha\beta}$ with 16×16 components, there always remains at least one massless four-dimensional Ramond-Ramond field with

4×4 components. Using two-component Weyl notation, this massless four-dimensional field splits into F_{ab} and $\tilde{F}_{a\dot{b}}$, and their complex conjugates, $\bar{F}_{\dot{a}\dot{b}}$ and $\bar{\tilde{F}}_{\dot{a}b}$ (a,b=1 or 2).

So from the analysis of the previous section, the massless four-dimensional Ramond-Ramond fields always contain the following fields for $n=0$ to ∞:

$$F_{(n)ab}, E_{(n)\dot{a}b}, D_{(n)a\dot{b}}, C_{(n)\dot{a}\dot{b}}, \quad \bar{F}_{(n)\dot{a}\dot{b}}, \bar{E}_{(n)a\dot{b}}, \bar{D}_{(n)\dot{a}b}, \bar{C}_{(n)ab}, \tag{4.1}$$

$$\tilde{F}_{(n)a\dot{b}}, \tilde{E}_{(n)\dot{a}\dot{b}}, \tilde{D}_{(n)ab}, \tilde{C}_{(n)\dot{a}b}, \quad \bar{\tilde{F}}_{(n)\dot{a}b}, \bar{\tilde{E}}_{(n)ab}, \bar{\tilde{D}}_{(n)\dot{a}\dot{b}}, \bar{\tilde{C}}_{(n)a\dot{b}}.$$

The free action for these four-dimensional massless fields is easily obtained from (3.22)by ignoring 12 of the 16 components of each ten- dimensional spinor. The resulting action is:

$$\begin{aligned}
\mathcal{S} = \sum_{n=0}^{\infty} [2C_{(n)\dot{a}\dot{b}}\partial_\mu\partial^\mu(\partial^{a\dot{a}}\bar{E}^{\dot{b}}_{(n)a} + (-1)^n\partial^{b\dot{b}}\bar{D}^{\dot{a}}_{(n)b})\\
-2F^{ab}_{(n)}((-1)^n\partial_{b\dot{b}}\bar{E}^{\dot{b}}_{(n)a} + \partial_{a\dot{a}}\bar{D}^{\dot{a}}_{(n)b})\\
-(F^{ab}_{(n)} + \partial^{a\dot{a}}E^{b}_{(n)\dot{a}} + (-1)^n\partial^{b\dot{b}}D^{a}_{(n)\dot{b}} - (-1)^n\partial^{a\dot{a}}\partial^{b\dot{b}}C_{(n)\dot{a}\dot{b}})\\
(F_{(n+1)ab} + (-1)^n\partial_{b\dot{c}}D^{\dot{c}}_{(n+1)a} - \partial_{a\dot{c}}E^{\dot{c}}_{(n+1)b} + (-1)^n\partial_{a\dot{c}}\partial_{b\dot{d}}C^{\dot{c}\dot{d}}_{(n+1)})]\\
\\
+\sum_{n=0}^{\infty}[2\tilde{C}_{(n)\dot{a}b}\partial_\mu\partial^\mu(\partial^{a\dot{a}}\bar{\tilde{E}}^{b}_{(n)a} + (-1)^n\partial^{b\dot{b}}\bar{\tilde{D}}^{\dot{a}}_{(n)\dot{b}})\\
-2\tilde{F}^{a\dot{b}}_{(n)}((-1)^n\partial_{b\dot{b}}\bar{\tilde{E}}^{b}_{(n)a} + \partial_{a\dot{a}}\bar{\tilde{D}}^{\dot{a}}_{(n)\dot{b}})\\
-(\tilde{F}^{a\dot{b}}_{(n)} + \partial^{a\dot{a}}\tilde{E}^{\dot{b}}_{(n)\dot{a}} + (-1)^n\partial^{b\dot{b}}\tilde{D}^{a}_{(n)b} - (-1)^n\partial^{a\dot{a}}\partial^{b\dot{b}}\tilde{C}_{(n)\dot{a}b})\\
(\tilde{F}_{(n+1)a\dot{b}} + (-1)^n\partial_{c\dot{b}}\tilde{D}^{c}_{(n+1)a} - \partial_{a\dot{c}}\tilde{E}^{\dot{c}}_{(n+1)\dot{b}} + (-1)^n\partial_{a\dot{c}}\partial_{d\dot{b}}\tilde{C}^{\dot{c}d}_{(n+1)})]\\
+ \textit{complex conjugate}.
\end{aligned} \tag{4.2}$$

Note that tilded and un-tilded fields do not couple in (4.2), which is related to the non-coupling of hypermultiplets and vector multiplets in low-energy N=2 actions.

After gauge-fixing

$$E^{\dot{a}b}_{(n)} = D^{a\dot{b}}_{(n)} = C^{\dot{a}\dot{b}}_{(n)} = \tilde{E}^{\dot{a}\dot{b}}_{(n)} = \tilde{D}^{ab}_{(n)} = \tilde{C}^{\dot{a}b}_{(n)} = 0 \tag{4.3}$$

as in (3.25), the equations of motion are

$$0 = F^{ab}_{(1)} = -F^{ab}_{(3)} = F^{ab}_{(5)} = ..., \quad 0 = \tilde{F}^{a\dot{b}}_{(1)} = -\tilde{F}^{a\dot{b}}_{(3)} = \tilde{F}^{a\dot{b}}_{(5)} = ..., \tag{4.4}$$

$$F^{ab}_{(0)} = -F^{ab}_{(2)} = F^{ab}_{(4)} = ..., \quad \tilde{F}^{a\dot{b}}_{(0)} = -\tilde{F}^{a\dot{b}}_{(2)} = \tilde{F}^{a\dot{b}}_{(4)} = ..., \tag{4.5}$$

$$\partial_{a\dot{a}} F^{ab}_{(0)} = \partial_{b\dot{b}} F^{ab}_{(0)} = 0, \quad \partial_{a\dot{a}} \tilde{F}^{a\dot{b}}_{(0)} = \partial_{b\dot{b}} \tilde{F}^{a\dot{b}}_{(0)} = 0. \tag{4.6}$$

Using the vector notation,

$$F^{ab}_{(0)} = F_{(0)}\epsilon^{ab} + F^{mn}_{(0)}\sigma^{ab}_{mn}, \quad \tilde{F}^{a\dot{b}}_{(0)} = \tilde{F}^{m}_{(0)}\sigma^{a\dot{b}}_{m}, \tag{4.7}$$

one gets the equations of motion from (4.6):

$$\partial_m F_{(0)} = \partial_m F^{mn}_{(0)} = \partial_m \tilde{F}^{m}_{(0)} = 0, \tag{4.8}$$

$$\epsilon_{mnpq}\partial^m F^{np}_{(0)} = \epsilon_{mnpq}\partial^m \tilde{F}^{n}_{(0)} = 0. \tag{4.9}$$

(Note that $F^{mn}_{(0)}$ is real, while $F_{(0)}$ and $\tilde{F}^{m}_{(0)}$ are complex.)

Equations (4.9)imply that $F^{mn}_{(0)}$ and $\tilde{F}^{m}_{(0)}$ can be expressed in terms of a real U(1) gauge field A_m and a complex scalar y as

$$F^{mn}_{(0)} = \partial^m A^n - \partial^n A^m, \quad \tilde{F}^{m}_{(0)} = \partial^m y.$$

Equations (4.8)imply that A_m and y propagate on-shell and that $F_{(0)}$ is a constant. Together with the NS-NS graviton, axion, and dilaton, the R-R fields A_m, y, and $\bar{y}$ form the bosonic degrees of freedom of an N=2 supergravity and dilaton multiplet.

Although all previous descriptions of the four-dimensional R-R U(1) gauge field assumed an action of the form $\int d^4x F^{mn}F_{mn}$, it is seen from (4.2)that the action coming from closed superstring field theory is completely different and requires an infinite number of fields. As will now be shown, the action of (4.2)is manifestly invariant under the electromagnetic duality transformation, $F^{mn} \to \epsilon^{mnpq}F_{pq}$, whereas for the usual $\int d^4x F^{mn}F_{mn}$ action, only the equations of motion are invariant.

In spinor notation, the electromagnetic duality transformation is

$$F^{ab} \to iF^{ab}, \quad \bar{F}^{\dot{a}\dot{b}} \to -i\bar{F}^{\dot{a}\dot{b}}, \tag{4.10}$$

or in its continuous version,

$$F^{ab} \to e^{i\theta}F^{ab}, \quad \bar{F}^{\dot{a}\dot{b}} \to e^{-i\theta}\bar{F}^{\dot{a}\dot{b}} \tag{4.11}$$

where θ is an arbitrary real constant.

It is easy to see that the action of (4.2)is invariant under $F^{ab}_{(0)} \to e^{i\theta} F^{ab}_{(0)}$ if the un-tilded fields transform as

$$F^{ab}_{(2n)} \to e^{i\theta} F^{ab}_{(2n)}, \quad E^{\dot{a}b}_{(2n)} \to e^{i\theta} E^{\dot{a}b}_{(2n)}, \tag{4.12}$$

$$D^{a\dot{b}}_{(2n)} \to e^{i\theta} D^{a\dot{b}}_{(2n)}, \quad C^{\dot{a}\dot{b}}_{(2n)} \to e^{i\theta} C^{\dot{a}\dot{b}}_{(2n)},$$

$$F^{ab}_{(2n+1)} \to e^{-i\theta} F^{ab}_{(2n+1)}, \quad E^{\dot{a}b}_{(2n+1)} \to e^{-i\theta} E^{\dot{a}b}_{(2n+1)},$$

$$D^{a\dot{b}}_{(2n+1)} \to e^{-i\theta} D^{a\dot{b}}_{(2n+1)}, C^{\dot{a}\dot{b}}_{(2n+1)} \to e^{-i\theta} C^{\dot{a}\dot{b}}_{(2n+1)},$$

and the tilded fields are left unchanged. So, unlike the $\int d^4x F^{mn} F_{mn}$ action, the action of (4.2)is manifestly invariant under the electromagnetic duality transformation of (4.10).

5. Conclusions

In this paper, I have derived the massless Ramond-Ramond contribution to the free action using closed superstring field theory. This action contains an infinite number of fields and, after compactification to four dimensions, is manifestly invariant under electromagnetic duality transformations.

There are various possible generalizations of this work. One generalization would be to combine the NS-NS, NS-R, R-NS, and R-R contributions into a manifestly spacetime-supersymmetric action. After compactification to four dimensions, this is already possible using a closed superstring field theory[16] based on the new spacetime-supersymmetric description of the superstring[17]. (In fact, this was how the action of (4.2)was originally discovered.)

A second generalization would be to construct the complete interacting contribution of the R-R fields. It would be very interesting to see if manifest electromagnetic duality is preserved at the interacting level. It would also be interesting to compute the dilaton coupling of R-R fields and see if standard arguments[2]based on $\int d^4x F^{mn} F_{mn}$-type actions need to be revised.

A third generalization would be to extend the manifest electromagnetic duality to SL(2,R) duality by including interactions with scalar fields. As discussed in reference [6], SL(2,R) duality is naturally manifest in an action coming from five-branes, as opposed to T-duality, which is naturally manifest in an action coming from strings. However, in this case, both T-duality and SL(2,R) duality would be manifest in an action coming from strings.

Acknowledgements: I would like to thank Ashok Das, Martin Roĉek, Warren Siegel, and Barton Zwiebach for useful conversations. This work was financially supported by the Brazilian Conselho Nacional de Pesquisa.

References

[1] A. Sen, Int. J. Mod. Phys. A9 (1994) 3707;
C. Hull and P. Townsend, Nucl. Phys. B438 (1995) 109.

[2] E. Witten, Nucl. Phys. B443 (1995) 85.

[3] A. Sen and B. Zwiebach, private communication.

[4] N. Marcus and J.H. Schwarz, Phys. Lett. B115 (1982) 111.

[5] S. Deser and C. Teitelboim, Phys. Rev. D13 (1976) 1592;
M. Henneaux and C. Teitelboim, Phys. Lett. B206 (1988) 650.

[6] A. Sen and J.H. Schwarz, Nucl. Phys. B411 (1994) 35.

[7] P. Pasti, D. Sorokin, and M. Tonin, "Duality Symmetric Actions with Manifest Space-Time Supersymmetry", hep-th 9506109.

[8] D. Sorokin, private communication.

[9] W. Siegel, Int. J. Mod. Phys. A6 (1991) 3997;
N. Berkovits, M.T. Hatsuda, and W. Siegel, Nucl. Phys. B371 (1991) 434.

[10] W. Siegel, Phys. Lett. B151 (1985) 396;
W. Siegel and B. Zwiebach, Nucl. Phys. B263 (1986) 105.

[11] B. McClain, Y. Wu, and F. Yu, Nucl. Phys. B343 (1990) 689;
I. Martin and A. Restuccia, Phys. Lett. B323 (1994) 311;
F.P. Devecchi and M. Henneaux, "Covariant path integral for chiral p-forms", hep-th 9603031.

[12] D. Friedan, E. Martinec, and S. Shenker, Nucl. Phys. B271 (1986) 93.

[13] E. Witten, Nucl. Phys. B276 (1986) 291.

[14] I.Ya. Aref'eva and P.B. Medvedev, Phys. Lett. B202 (1988) 510.

[15] B. Zwiebach, Nucl. Phys. B390 (1993) 33.

[16] N. Berkovits, work in progress.

[17] N. Berkovits, "A New Description of the Superstring", to appear in proceedings of the VIII J.A. Swieca summer school, hep-th 9604123;
N. Berkovits, Nucl. Phys. B459 (1996) 439.